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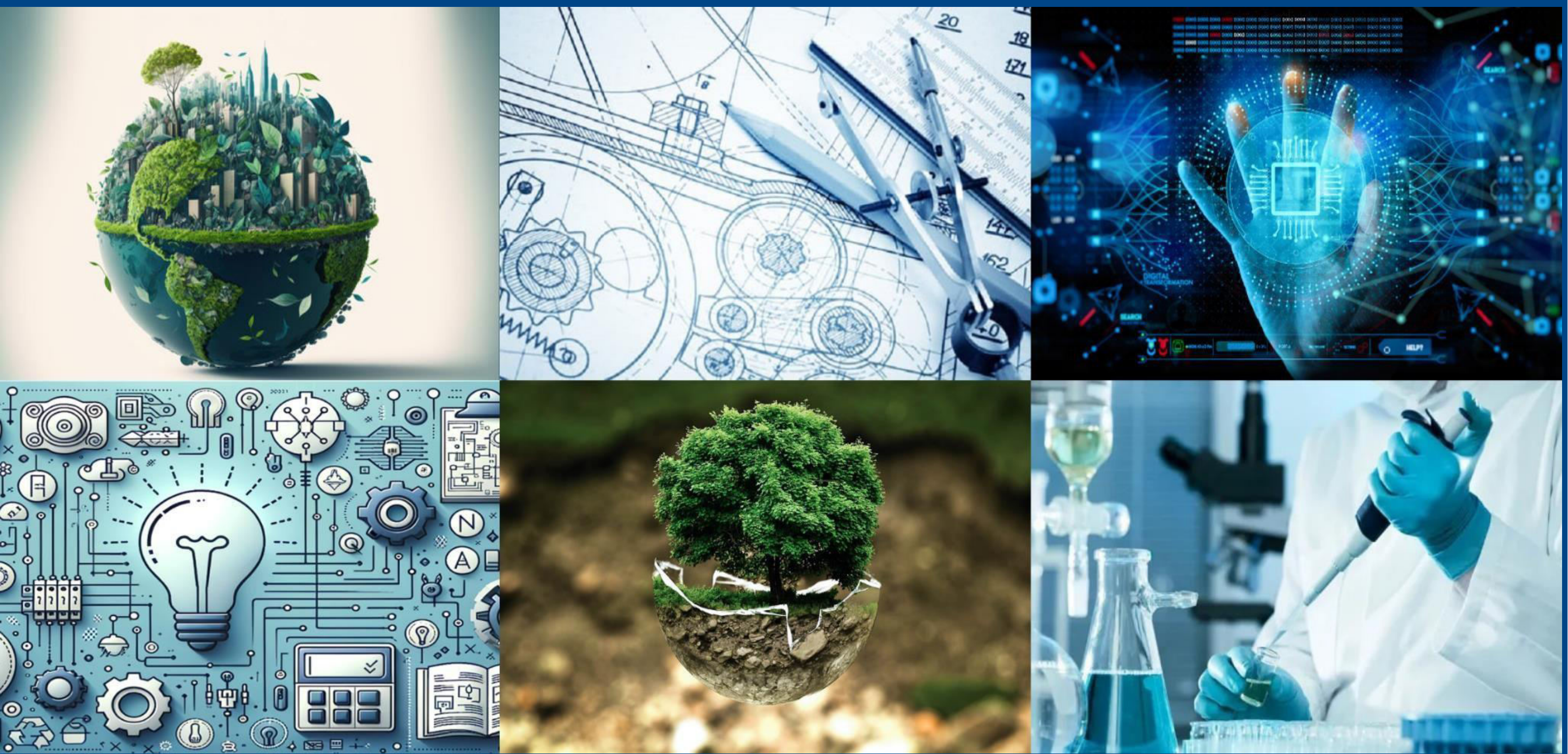

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Existence and Uniqueness of Solutions to the Wave Equation with Mixed Boundary Conditions

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ABSTRACT: We investigate the existence and uniqueness of solutions to the classical wave equation subject to mixed boundary conditions on a bounded domain. The problem is formulated within an appropriate functional framework using Sobolev spaces, allowing for a combination of Dirichlet and Neumann boundary conditions on disjoint portions of the boundary. By employing energy methods and variational techniques, we establish the well-posedness of the initial–boundary value problem. In particular, we prove the existence of weak solutions and demonstrate their uniqueness under suitable regularity assumptions on the initial data and forcing terms. Additional regularity results are obtained, showing continuous dependence of solutions on the given data. These results ensure the stability and physical relevance of the model and provide a rigorous foundation for further analytical and numerical studies of wave propagation under mixed boundary constraints.

KEYWORDS: Wave equation; mixed boundary conditions; existence and uniqueness; weak solutions; energy estimates; Sobolev spaces; initial–boundary value problem

I. INTRODUCTION

The wave equation is one of the fundamental partial differential equations in mathematical physics, arising naturally in the modeling of vibrating strings, elastic membranes, acoustic waves, and electromagnetic phenomena. In its simplest form, the wave equation describes the propagation of disturbances through a medium over time and captures the balance between inertia and restoring forces. Because of its broad applicability, understanding the well-posedness of the wave equation—namely the existence, uniqueness, and stability of its solutions—has long been a central topic in the theory of partial differential equations.

In practical applications, the behavior of waves is strongly influenced by boundary conditions imposed on the spatial domain. These boundary conditions represent physical constraints such as fixed endpoints, free boundaries, or energy dissipation at the boundary. Among the most common types are Dirichlet boundary conditions, which prescribe the value of the solution on the boundary, and Neumann boundary conditions, which prescribe the normal derivative and thus the flux across the boundary. In many realistic situations, different parts of the boundary are subject to different physical constraints, leading naturally to mixed boundary conditions. For example, a vibrating string may be fixed at one end while the other end is free, or a membrane may be partially clamped and partially insulated. Such mixed boundary conditions introduce additional analytical challenges compared to purely Dirichlet or Neumann cases.

The study of existence and uniqueness of solutions to the wave equation with mixed boundary conditions is essential for ensuring that the mathematical model is meaningful and predictive. Existence guarantees that a solution describing the physical phenomenon actually exists, while uniqueness ensures that the solution is determined solely by the initial and boundary data, reflecting determinism in the underlying physical system. Without uniqueness, multiple solutions could correspond to the same physical setup, undermining the reliability of the model.

From a mathematical perspective, mixed boundary conditions require careful treatment of function spaces and boundary traces. The natural framework for analyzing the wave equation is typically based on Sobolev spaces, where weak solutions can be defined and studied using variational methods. Energy estimates play a crucial role in this analysis, allowing one to control the growth of solutions in time and to establish uniqueness through continuous dependence on initial data. In particular, the interaction between different boundary conditions must be handled delicately to ensure that the associated energy functional remains well defined and nonnegative.

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This work focuses on establishing existence and uniqueness results for the wave equation subject to mixed boundary conditions on a bounded spatial domain. By employing functional analytic techniques, such as the Galerkin method and energy methods, we demonstrate that under suitable assumptions on the initial data and boundary conditions, the problem is well posed in an appropriate Sobolev space setting. These results not only provide a rigorous mathematical foundation for the wave equation with mixed boundary conditions but also support its use in modeling a wide range of physical and engineering problems where heterogeneous boundary effects are unavoidable.

II. DIFFERENTIATION AND INTEGRATION IN BANACH SPACES

Before moving on, we quickly go over the fundamentals of calculus in Banach spaces. C(X, Y) is the set of continuous functions from X to Y, and L(X, Y) is the set of (bounded) linear functions. Assume that X and Y are two Banach spaces. An open subset of X is denoted by U. A differentiable function at every point x in U is defined as a function F: U → Y for which there exists a linear function dF(x) in the set L(X, Y) such that

$F(x + u) = F(x) + dF(x)\, u + o(u)$,

Where o, O are the Landau symbols. The linear map dF(x) is called derivative of F at x. If F is differentiable for all $x \in U$ we call F differentiable. In this case we get a map

$dF : U \to L(X, Y)$

$x \to dF(x)$.

If dF is continuous, we call F continuously differentiable and write $F \in C^1(U, Y)$. Let

$$Y = \prod_{j=1}^{m} Y_j$$

And let $F : X \to Y$ be given by $F = (F_1, \ldots, F_m)$ with $F_j : X \to Y_i$. Then $F \in C^1(X, Y)$ if and only if $F_j \in C^1(X, Y_j)$, $1 \le j \le m$, and in this case $dF = (dF_1, \ldots, dF_m)$. Similarly, if X = Qm i=1 Xi , then one can define the partial derivative $\partial_i F \in L(X_i, Y)$, which is the derivative of F considered as a function of the i-th variable alone (the other variables being fixed). We have

$$dF\, v = \sum_{i=1}^{n} \partial_i F\, v_i, \quad v = (v_1, \ldots, v_n) \in X$$

and $F \in C^1(X, Y)$ if and only if all partial derivatives exist and are continuous.

In the case of $X = R^m$ and $Y = R^n$,the matrix representation of dF with respect to the canonical basis in R^m and R^n is given by the partial derivatives $\partial_i F_j(x)$ and is called Jacobi matrix of F at x. We can iterate the procedure of differentiation and write $F \in C^r(U, Y)$, $r \ge 1$, if the r-th derivative of F, $d^r F$ (i.e., the derivative of the (r − 1)-th derivative of F), exists and is continuous. Finally, we set

$$C^\infty(U, Y) = \bigcap_{r \in \mathbb{N}} C^r(U, Y)$$

and, for notational convenience, $C^0(U, Y) = C(U, Y)$ and $d^0 F = F$. It is often necessary to equip $C^r(U, Y)$ with a norm. A suitable choice is

$$|F| = \max_{0 \le j \le r} \sup_{x \in U} |d^j F(x)|.$$

The set of all r times continuously differentiable functions for which this norm is finite forms a Banach space which is denoted by $C^r_b(U, Y)$.

If F is bijective and F, F^{-1} are both of class C^r, $r \ge 1$, then F is called a diffeomorphism of class C^r. Note that if $F \in L(X, Y)$, then dF(x) = F (independent of x) and $d^r F(x) = 0$, $r > 1$. For the composition of mappings we note the following result (which is easy to prove).

Lemma (Chain rule) Let $F \in C^r(X, Y)$ and $G \in C^r(Y, Z)$, $r \ge 1$. Then $G \circ F \in C^r(X, Z)$ and

$d(G \circ F)(x) = dG(F(x)) \circ dF(x)$, $x \in X$.

In particular, if $\lambda \in Y^*$ is a linear functional, then $d(\lambda \circ F) = d\lambda \circ dF = \lambda \circ dF$. In addition, we have the following mean value theorem.

Theorem (Mean value) Suppose $U \subseteq X$ and $F \in C^1(U, Y)$. If U is convex, then

$$|F(x) - F(y)| \le M|x - y|, \qquad M = \max_{0 \le t \le 1} |dF((1 - t)x + ty)|$$

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Conversely, (for any open U) if
|F(x) − F(y)| ≤ M|x − y|, x, y ∈ U,
then

$$\sup_{x\in U} |dF(x)| \le M.$$

Proof. Abbreviate f(t) = F((1 − t)x + ty), 0 ≤ t ≤ 1, and hence df(t) = dF((1 − t)x + ty)(y − x) implying |df(t)| ≤ M˜ = M|x − y|. For the first part it suffices to show
φ(t) = |f(t) − f(0)| − (M˜ + δ)t ≤ 0
for any δ > 0. Let t0 = max{t ∈ [0, 1]|φ(t) ≤ 0}. If t0 < 1 then

$$\begin{aligned}\phi(t_0+\varepsilon) &= |f(t_0+\varepsilon)-f(t_0)+f(t_0)-f(0)|-(\tilde{M}+\delta)(t_0+\varepsilon)\\ &\le |f(t_0+\varepsilon)-f(t_0)|-(\tilde{M}+\delta)\varepsilon+\phi(t_0)\\ &\le |df(t_0)\varepsilon+o(\varepsilon)|-(\tilde{M}+\delta)\varepsilon\\ &\le (\tilde{M}+o(1)-\tilde{M}-\delta)\varepsilon=(-\delta+o(1))\varepsilon\le 0,\end{aligned}$$

for ε ≥ 0, small enough. Thus t0 = 1. To prove the second claim suppose there is an x0 ∈ U such that |dF(x0)| = M + δ, δ > 0. Then we can find an e ∈ X, |e| = 1 such that |dF(x0)e| = M + δ and hence

$$\begin{aligned}M\varepsilon &\ge |F(x_0+\varepsilon e)-F(x_0)| = |dF(x_0)(\varepsilon e)+o(\varepsilon)|\\ &\ge (M+\delta)\varepsilon-|o(\varepsilon)|>M\varepsilon\end{aligned}$$

since we can assume |o(ε)| < εδ for ε > 0 small enough, a contradiction. As an immediate consequence we obtain.
Corollary Suppose U is a connected subset of a Banach space X. A mapping F ∈ C 1 (U, Y) is constant if and only if dF = 0. In addition, if F1,2 ∈ C 1 (U, Y) and dF1 = dF2, then F1 and F2 differ only by a constant.
Next we want to look at higher derivatives more closely. Let

$$X = \prod_{i=1}^{m} X_i,$$

then F : X → Y is called multilinear if it is linear with respect to each argument. It is not hard to see that F is continuous if and only if

$$|F| = \sup_{x:\prod_{i=1}^{m}|x_i|=1} |F(x_1,\ldots,x_m)| < \infty$$

If we take n copies of the same space, the set of multilinear functions F : Xn → Y will be denoted by L n (X, Y). A multilinear function is called symmetric provided its value remains unchanged if any two arguments are switched. With the norm from above it is a Banach space and in fact there is a canonical isometric isomorphism between L n (X, Y) and L(X,L n−1 (X, Y)) given by F : (x1, . . . , xn) → F(x1, . . . , xn) maps to x1 → F(x1, .). In addition, note that to each F ∈ Ln (X, Y) we can assign its polar form F ∈ C(X, Y) using F(x) = F(x, . . . , x), x ∈ X. If F is symmetric it can be reconstructed from its polar form using

$$F(x_1,\ldots,x_n) = \frac{1}{n!}\partial_{t_1}\cdots\partial_{t_n}F(\sum_{i=1}^{n} t_i x_i)|_{t_1=\cdots=t_n=0}$$

Moreover, the r-th derivative of F ∈ C r (X, Y) is symmetric since,

$$d^r F_x(v_1,\ldots,v_r) = \partial_{t_1}\cdots\partial_{t_r}F(x+\sum_{i=1}^{r} t_i v_i)|_{t_1=\cdots=t_r=0}$$

Where the order of the partial derivatives can be shown to be irrelevant.

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Now we turn to integration. We will only consider the case of mappings f : I → X where I = [a, b] ⊂ R is a compact interval and X is a Banach space. A function f : I → X is called simple if the image of f is finite,

$$f(I) = \{x_i\}_{i=1}^{n}$$

and if each inverse image $f^{-1}(xi)$, 1 ≤ i ≤ n is a Borel set. The set of simple functions S(I, X) forms a linear space and can be equipped with the sup norm. The corresponding Banach space obtained after completion is called the set of regulated functions R(I, X). Observe that C(I, X) ⊂ R(I, X). In fact, consider

$$f_n = \sum_{i=0}^{n-1} f(t_i)\chi_{[t_i,t_{i+1})}$$

∈ S(I, X), where

$$t_i = a + i\frac{b-a}{n}$$

and χ is the characteristic function. Since f ∈ C(I, X) is uniformly continuous, we infer that fn converges uniformly to f. For f ∈ S(I, X) we can define a linear map R : S(I, X) → X by

$$\int_a^b f(t)dt = \sum_{i=1}^{n} x_i \mu(f^{-1}(x_i)),$$

where μ denotes the Lebesgue measure on I. This map satisfies

$$\int_a^b f(t)dt \leq |f|(b-a).$$

and hence it can be extended uniquely to a linear map R : R(I, X) → X with the same norm (b − a). We even have

$$\int_a^b f(t)dt \leq \int_a^b |f(t)|dt.$$

In addition, if λ ∈ X∗ is a continuous linear functional, then

$$\lambda(\int_a^b f(t)dt) = \int_a^b \lambda(f(t))dt, \qquad f \in R(I,X).$$

We use the usual conventions

$$\int_{t_1}^{t_2} f(s)ds = \int_a^b \chi_{(t_1,t_2)}(s) f(s)ds \text{ and } \int_{t_2}^{t_1} f(s)ds = -\int_{t_1}^{t_2} f(s)ds.$$

If I ⊆ R, we have an isomorphism L(I, X) ≡ X and if F : I → X we will write F˙(t) in stead of dF(t) if we regard dF(t) as an element of X. In particular, if f ∈ C(I, X), then

$$F(t) = \int_a^t f(s)ds \in C^1(I,X)$$

and F˙(t) = f(t) as can be seen from

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$$|\int_a^{t+\varepsilon} f(s)ds - \int_a^t f(s)ds - f(t)\varepsilon| = |\int_t^{t+\varepsilon} (f(s)-f(t))ds| \le |\varepsilon| \sup_{s\in[t,t+\varepsilon]} |f(s)-f(t)|.$$

This even shows that

$$F(t) = F(a) + \int_a^t (\dot{F}(s))ds$$

for any F ∈ C^1 (I, X).

III. CONTRACTION PRINCIPLES

A fixed point of a mapping F : C ⊆ X → C is an element x ∈ C such that F(x) = x. Moreover, F is called a contraction if there is a contraction constant θ ∈ [0, 1) such that
|F(x) − F(˜x)| ≤ θ|x − x˜|, x, x˜ ∈ C.
Note that a contraction is continuous. We also recall the notation F n (x) = F(F n−1 (x)), F 0 (x) = x.
Theorem (Contraction principle) Let C be a closed subset of a Banach space X and let F : C → C be a contraction, then F has a unique fixed point x ∈ C such that

$$|F^n(x) - \overline{x}| \le \frac{\theta^n}{1-\theta}|F(x) - x|, \qquad x \in C.$$

Proof. If x = F(x) and ˜x = F(˜x), then |x − x˜| = |F(x) − F(˜x)| ≤ θ|x − x˜| shows that there can be at most one fixed point. Concerning existence, fix x0 ∈ C and consider the sequence xn = F ^{n} (x0). We have
xn+1 − xn| ≤ θ|xn − xn−1| ≤ · · · ≤ θ ^{n} |x1 − x0|
and hence by the triangle inequality (for n > m)

$$\begin{aligned} |x_n - x_m| &\le \sum_{j=m+1}^{n} |x_j - x_{j-1}| \le \theta^m \sum_{j=0}^{n-m-1} \theta^j |x_1 - x_0| \\ &\le \frac{\theta^m}{1-\theta}|x_1 - x_0|. \end{aligned}$$

Thus xn is Cauchy and tends to a limit$\bar{x}$. Moreover

$$|F(\overline{x}) - \overline{x}| = \lim_{n\to\infty} |x_{n+1} - x_n| = 0$$

shows that $\bar{x}$ is a fixed point and the estimate follows after taking the limit n → ∞ in.
Next, we want to investigate how fixed points of contractions vary with respect to a parameter. Let U ⊆ X, V ⊆ Y be open and consider F : $\overline{U}$ × V → U. The mapping F is called a uniform contraction if there is a θ ∈ [0, 1) such that

$$|F(x,y) - F(\tilde{x},y)| \le \theta|x - \tilde{x}|, \quad x,\tilde{x} \in \overline{U},\ y \in V.$$

Theorem (Uniform contraction principle) Let U, V be open subsets of Banach spaces X, Y , respectively. Let F : $\overline{U}$ × V → U be a uniform contraction and denote by $\bar{x}$ (y) ∈ U the unique fixed point of F(., y). If F ∈ C r (U × V, U), r ≥ 0, then $\bar{x}$ (.) ∈ C r (V, U).
Proof. Let us first show that x(y) is continuous. From

$$\begin{aligned} |\overline{x}(y+v) - \overline{x}(y)| &= |F(\overline{x}(y+v), y+v) - F(\overline{x}(y), y+v) \\ &\quad + F(\overline{x}(y), y+v) - F(\overline{x}(y), y)| \\ &\le \theta|\overline{x}(y+v) - \overline{x}(y)| + |F(\overline{x}(y), y+v) - F(\overline{x}(y), y)| \end{aligned}$$

we infer

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$$|\overline{x}(y+v)-\overline{x}(y)| \leq \frac{1}{1-\theta}|F(\overline{x}(y),y+v)-F(\overline{x}(y),y)|$$

and hence $\bar{x}(y) \in C(V, U)$. Now let r = 1 and let us formally differentiate $\bar{x}(y) = F(\bar{x}(y), y)$ with respect to y,

$$d\,\overline{x}(y) = \partial_x F(\overline{x}(y), y) d\,\overline{x}(y) + \partial_y F(\overline{x}(y), y).$$

Considering this as a fixed point equation T(x 0 , y) = x 0 , where

$$T(.,y) : \mathcal{L}(Y,X) \to$$

$$\mathcal{L}(Y,X),\ x' \mapsto \partial_x F(\overline{x}(y),y)x' + \partial_y F(\overline{x}(y),y)$$

is a uniform contraction since we have $|\partial xF(\bar{x}(y), y)| \leq \theta$ by Theorem. Hence we get a unique continuous solution $\bar{x}'(y)$. It remains to show

$$\overline{x}(y+v) - \overline{x}(y) - \overline{x}'(y)v = o(v)$$

Let us abbreviate $u = \bar{x}(y + v) - \bar{x}(y)$, then using and the fixed point property of $\bar{x}(y)$ we see

$$(1 - \partial_x F(\overline{x}(y), y))(u - \overline{x}'(y)v) =$$

$$= F(\overline{x}(y) + u, y + v) - F(\overline{x}(y), y) - \partial_x F(\overline{x}(y), y)u - \partial_y F(\overline{x}(y), y)v = o(u) + o(v)$$

since F ∈ C 1 (U×V, U) by assumption. Moreover, $|(1-\partial xF(\bar{x}(y), y))-1| \leq (1-\theta)-1$ and u = O(v) (by implying $u - \bar{x}'(y)v = o(v)$ as desired. Finally, suppose that the result holds for some r − 1 ≥ 1. Thus, if F is C r , then $\bar{x}(y)$ is at least C r−1 and the fact that d $\bar{x}(y)$ satisfies implies $\bar{x}(y) \in$ C r (V, U). As an important consequence we obtain the implicit function theorem.

Theorem (Implicit function) Let X, Y , and Z be Banach spaces and let U, V be open subsets of X, Y , respectively. Let F ∈ C r (U × V, Z), r ≥ 1, and fix (x0, y0) ∈ U × V . Suppose ∂xF(x0, y0) ∈ L(X, Z) is an isomorphism. Then there exists an open neighborhood U1 × V1 ⊆ U × V of (x0, y0) such that for each y ∈ V1 there exists a unique point (ξ(y), y) ∈ U1 × V1 satisfying F(ξ(y), y) = F(x0, y0). Moreover, the map ξ is in C r (V1, Z) and fulfills
$d\xi(y) = -(\partial xF(\xi(y), y))^{-1} \circ \partial yF(\xi(y), y)$.

Proof. Using the shift F → F − F(x0, y0) we can assume F(x0, y0) = 0. Next, the fixed points of $G(x, y) = x - (\partial xF(x0, y0))^{-1}F(x, y)$ are the solutions of F(x, y) = 0. The function G has the same smoothness properties as F and since |∂xG(x0, y0)| = 0, we can find balls U1 and V1 around x0 and y0 such that |∂xG(x, y)| ≤ θ < 1. Thus G(., y) is a uniform contraction and in particular, G(U1, y) ⊂ U1, that is, G : U1 × V1 → U1. The rest follows from the uniform contraction principle. Formula follows from differentiating F(ξ(y), y) = 0 using the chain rule.

Note that our proof is constructive, since it shows that the solution ξ(y) can be obtained by iterating $x - (\partial xF(x0, y0))^{-1}F(x, y)$. Moreover, as a corollary of the implicit function theorem we also obtain the inverse function theorem.

Theorem (Inverse function) Suppose F ∈ C r (U, Y), U ⊆ X, and let dF(x0) be an isomorphism for some x0 ∈ U. Then there are neighborhoods U1, V1 of x0, F(x0), respectively, such that F ∈ C r (U1, V1) is a diffeomorphism.

Proof. Apply the implicit function theorem to G(x, y) = y − F(x).

IV. CONCLUSION

In this work, we investigated the existence and uniqueness of solutions to the wave equation subject to mixed boundary conditions. By formulating the problem within an appropriate functional framework and employing energy methods, we established that, under suitable assumptions on the initial data and boundary terms, the problem is well posed. In particular, we showed that a weak solution exists in the natural energy space and that this solution is unique.

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The key ingredient in the analysis was the derivation of an energy identity (or inequality) adapted to the mixed boundary conditions, which allowed us to control the solution in terms of the initial data. Uniqueness followed from the same energy estimates by considering the difference of two solutions and showing that its energy must vanish identically. Existence was obtained via standard techniques such as Galerkin approximations and compactness arguments, ensuring convergence to a solution satisfying the equation in the weak sense.

These results confirm that mixed boundary conditions—combining, for example, Dirichlet and Neumann conditions—do not compromise the fundamental well-posedness of the wave equation, provided the boundary conditions are compatible with the energy framework. The analysis presented here lays a rigorous foundation for further studies, including higher regularity of solutions, numerical approximations, and extensions to more general domains or variable coefficients.

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